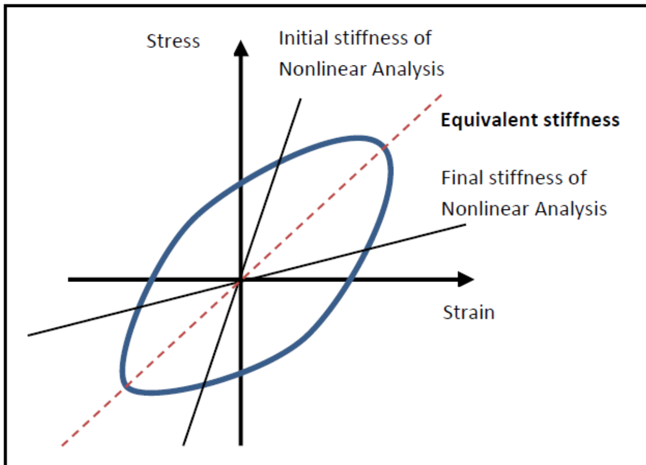
RSL-III-2D Local Seismic Response

Theoretical Backround

1. Introduction and Equivalent Linear Elastic Approach (ELA)

RSL-III-2D is a finite element system for seismic analysis of soils and structures. A two dimensional nonlinear (linear equivalent model) including the superstructure (liners, frames) finite element computer program has been developed (RSLIII-2D). The soil domain is analysed under the assumption of of plane strain condition. The materials in the various layers of the soil domain are modeled using the equivalent linear model (ELA). Basic idea of ELA is to solve the non-linear problem using *linear analysis* with a *reduced secant stiffness*, so that ELA can simulate the response at the cycle of the largest amplitude. To obtain the proper stiffness the linear analysis is repeatedly solved until the stiffness and the maximum shear strain response at each layer satisfy the material relationship between stiffness and the shear strain (namely, *G*-γ curve). Therefore ELA well simulates the response for the maximum magnitude cycle.



1. Set the possible stiffness K_i
2. Do dynamic linear analysis and obtain the maximum shear strain
3. Evaluate the stiffness K_{i+1} for the maximum shear strain from G-γ curve.
4. If $K_{i+1}= K_i$ then exit; othervise repeat step 2 with new stiffness K_{i+1}

Figure 1: Concept of ELA

Nevertheless, the response of the other cycle has error to the response of the NLA since stiffness used is the stiffness at the largest amplitude. The lateral and base boundary conditions for the computational soil domain are modeled using a modified Lysmer-Kuhlemeyer transmitting/absorbing boundary. A set of viscous normal and tangential to the soil boundaries can be used to implement these transmitting boundaries.

The Equivalent Linear analysis basically cycles through the entire earthquake record in an attempt to establish the correct the soil stiffness (*G-secant shear modulus*). During each pass through the earthquake record *the peak cyclic shear strains* are noted in each element. The soil stiffness (*G*) is then modified based on the peak cyclic shear strains. Once *G* has been adjusted, the entire process is repeated. Generally, five-to-ten iterations through the earthquake record is adequate to establish appropriate *G* values for each element. The number of iterations can be controlled with a user specified parameter. The default value in RSL-2D is set to 10 (ten) iterations.

When subject to a cyclic load, the soil usually exhibits a non-linear hysteretic stress-strain behaviour that can be approaximated with equivalent linear soil properties, such as secant shear modulus *G* and the damping ratio ξ which represent the inclination and the width of of the hysteretic loop respectively. According to the linear approach, *G* and ξ are constant for each soil element starting from the initial estimated values, they are updated in subsequent iterations to be consistent with the level of strain induced in each finite element. The procedure can be summarized as follows:

1. Initialize the values of $G^{(i)}$ and $\xi^{(i)}$ at their small strain values.

2. Compute the ground response, and get the amplitudes of maximum shear strain γ_{max} from the time-histories of shear strain in each soil finite element.

3. Determine the effective shear strain as:

$$\gamma_{eff}^{(i)} = R_{\gamma}\gamma_{\max}^{(i)} \qquad (1)$$

where R_γ is the ratio of the effective shear strain to maximum shear strain, which depends on the earthquake magnitude and is the same for all soil finite elements. The default value in RSL-III-2D is set to 0.65.

4. Calculate the new equivalent linear values $G^{(i+1)}$ and $\xi^{(i+1)}$ corresponding to the effective shear strain computed at step 3.

5. Repeat the iterations until the differences between the computed values of the shear modulus and damping ratio in two successive iterations is less than a specified error in all soil elements.

2. General time-history analysis algorithm

The finite element method allows irregular mesh with elements having different sizes and geometries to be used therefore it is very useful for modeling complex geometry and boundary conditions. The governing motion equation for dynamic response of a system can be expressed as:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{F} \tag{2}$$

where **M** is mass matrix, **C** is damping matrix, **K** is stiffness matrix, **F** is vector of loads (see section 2.3 of the present manual), **ü** is nodal acceleration vector, **u** is nodal velocity vector, **u** is nodal displacement vector assembled for the entire structure. Damping term is usually defined for each finite soil element as a linear combination (Rayleigh) of mass (**m**) and rigidity (**k**) as below.

$$\mathbf{c} = \alpha\mathbf{m} + \beta\mathbf{k} \tag{3}$$

where α and β are constant determined for each finite element as follows. The use of Rayleigh damping in this manner results in a frequency dependent damping applied to the system, with

$$\lambda = \frac{1}{2}\left(\frac{\alpha}{\omega} + \beta\omega\right) \tag{4}$$

where ω represents the natural circular frequency of the structures. Using this criterion one posbility to define the constants α and β is:

$$\begin{cases} \alpha = \lambda \omega_1 \\ \beta = \lambda / \omega_1 \end{cases} \tag{5}$$

where ω_1 represents the fundamental circular frequency of the structure (first mode of vibration). Such an approach is defined in the RSL-III-2D by means of chosing Single Frequency in Damping definition. Another definition for constants α and β are as follows:

$$\begin{cases} \alpha = 2\lambda \dfrac{\omega_1 \omega_2}{\omega_1 + \omega_2} \\ \beta = 2\lambda \dfrac{1}{\omega_1 + \omega_2} \end{cases} \tag{6}$$

where ω_2 represents the second frequency of the system

$$\omega_2 = n\omega_1 \tag{7}$$

where *n* is an odd integer. Such an approach is defined in the RSL-III-2D by means of chosing Double Frequency in Damping definition and defining a value for n_Damping on the analysis panel.

Remark: *The fundamnetal frequency of the system (soil structure) ω_1 is internally calculated by the program solving the following eigen value problem:*

$$\left(\mathbf{K} - \omega_1{}^2 \mathbf{M}\right)\Phi_1 = \mathbf{0} \tag{8}$$

where the first mode shape of vibration is $\boldsymbol{\Phi}_1$. It is important to note that the eigen value problem, stated above, is solved considering the boundary conditions of the structures associated to Stage 1 of the model definition where the classical fixed boundary conditions are considered. Furthermore, the user has the option to refine this approach by means of computing the eigen values during each iteration within the framework of Equivalent Linear Analysis approach summarised above.

In order to solve the Eq. (2) a direct integration method is applied in time domain. Time stepping is achieved using a direct Newmark method involving time-stepping parameters β and γ, the values of which determine the accuracy and stability characteristics of the algorithm. For the special case, adopted in the program RSL-III-2D, the method is identical to Crank-Nicolson method (Figure A):

$$\begin{cases} \beta = 1/4 \\ \gamma = 1/5 \end{cases} \tag{9}$$

Let $\mathbf{F}_i$, $\mathbf{u}_i$, $\overset{*}{\mathbf{u}}_i$, $\overset{**}{\mathbf{u}}_i$ represent conditions at time t=iΔt, where i=0,1,2,… nstep. Assuming that M, C, K, β, γ and Δt are constant and that F_i is known for all I, the following algorithm is used to obtain the values of $\mathbf{u}_i$, $\overset{*}{\mathbf{u}}_i$, $\overset{**}{\mathbf{u}}_i$ for all i>0.

$$\begin{cases} \beta = 1/4 \\ \gamma = 1/5 \end{cases}$$

1. Compute: $\mathbf{K}' = \mathbf{K} + \frac{\gamma}{\beta \Delta t}\mathbf{C} + \frac{1}{\beta \Delta t^2}\mathbf{M}$
2. Factorise **K'** to facilitate step 8.
3. Solve the linear equations: $\mathbf{M}\overset{**}{\mathbf{u}}_o = \mathbf{F}_0 - \mathbf{C}\overset{*}{\mathbf{u}}_o - \mathbf{K}\,\mathbf{u}_o$
4. Set i=0
5. Compute: $\mathbf{A}_i = \frac{1}{\beta \Delta t^2}\mathbf{u}_i + \frac{1}{\beta \Delta t}\overset{*}{\mathbf{u}}_i + \left(\frac{1}{2\beta} - 1\right)\overset{**}{\mathbf{u}}_i$
6. Compute: $\mathbf{B}_i = \frac{\gamma}{\beta \Delta t}\mathbf{u}_i - \left(1 - \frac{\gamma}{\beta}\right)\overset{*}{\mathbf{u}}_i - \left(1 - \frac{\gamma}{2\beta}\right)\Delta t\,\overset{**}{\mathbf{u}}_i$
7. Compute: $\mathbf{F}'_{i+1} = \mathbf{F}_{i+1} + \mathbf{M}\mathbf{A}_i + \mathbf{C}\mathbf{B}_i$
8. Solve the linear equations: $\mathbf{K}'\mathbf{u}_{i+1} = \mathbf{F}'_{i+1}$
9. Compute: $\overset{*}{\mathbf{u}}_{i+1} = \frac{\gamma}{\beta \Delta t}\mathbf{u}_{i+1} - \mathbf{B}_i$
10. Compute: $\overset{**}{\mathbf{u}}_{i+1} = \frac{1}{\beta \Delta t^2}\mathbf{u}_{i+1} - \mathbf{A}_i$
11. Increment *i* and repeat from step 5.

Figure A: Crank-Nicholson algorithm

2.1 Absorb and transmitting boundy conditions

In order for a 2D finite element finite domain to represent the response of an infinite field condition, the artificial reflection of seismic waves from side boundaries as well as from the underlying half-space (bottom part of the finite element model), should be minimized. The absorb and transmit boundary conditions provide the line segment with what is sometimes called a Lysmer-Kuhlemeyer (*L-K*) dashpot boundary. It is an artificial boundary condition that attempts to reproduce the infinite boundary behavior of the soil medium. It is important however to note that the effects of side boundaries can be readily minimized by increasing the extent of teh finite element mesh. That is to say that the absorb and transmit boundary absorb incoming shear and pressure waves as if the model was not actually bounded. The assumption of these boundaries is that the waves present in the system will propagate according to the soil material's shear and pressure wave velocities. The boundary therefore is constructed from two dampers at the external boundary, one perpendicular and the other tangential to the boundary orientation, whose damping coefficient is proportional to the wave velocities. The absorbent effect of soil layers and bedrock lying at the vertical and horizontal boundaries of the finite element model can be taken into consideration by putting viscous dashpots. Dashpot coefficients are proportional with the pressure and shear wave values of *the relevant soil layers at the boundaries* of 2D model. The implementation of these dampers involves a*dding damping at each of the nodes that make up the base and lateral sides* if the finite element model. To mathematically implement these dampers, the parts of the applicable element matrices have the transmitting boundary damping term *added to the diagonal terms in daming matrix of the structure*. This produces an adjustable force in the *x* and *y* dirrection proportional to the velocity of the specified nodes. The coefficients used for absorbing *P* and *S* waves added on the diagonal terms of the damping matrix are obtained as:

$$\begin{cases} c_p = \rho V_p L \\ c_s = \rho V_s L \end{cases} \tag{10}$$

Here c_p and c_s are damping coefficients that are used to absorb the energy of *P* (for *dirrection perpendicular to boundary*) and *S* (*dirrection parallel to boundary*) waves. V_p and V_s are *P* and *S* wave velocity of the relevant soil layers, ρ is the density of layers and *L* (*tributary width of the node*) is that length corresponding to half of the distance to the next node on both sides.

Remark 1*: In case the base of the model is fixed with restraints in both directions and especially when studied with strong ground motion acceleration which cause nonlinear behavior of the soil layers, the soil amplifications at the surface layers may reach to unrealistic high values during the numerical analyses. For this reason, viscous dashpots in two directions should be put at the base boundary of the model.*

The wave velocities are calculated using the following equations:

$$\begin{cases} V_s = \sqrt{\dfrac{E}{2(1+\nu)\rho}} \\ V_p = \sqrt{\dfrac{E(1+\nu)}{2(1-\nu)(1-2\nu)\rho}} \end{cases} \tag{11}$$

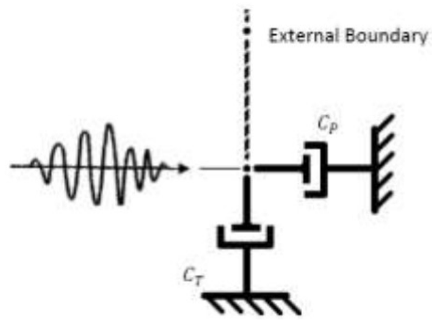


Figure 2: Damping coefficients. Absorb boundary conditions.

If an absorb or transmit boundary is applied on a line segment that borders elements with different material properties, an average value for modulus and density will be used in the damping coefficient computations.

Remark: *At each node along the base and lateral boundaries of the soil domain, a horizontal dashpot is set to transmit the shear waves at the base and compressive waves at the lateral boundaries respectively. The horizontal coefficient of the dashpot at the base is $\rho V_s L$ and the vertical coefficient is $\rho V_p L$, while that of dashpot on the lateral boundaries are $\rho V_p L$ for horizontal direction and $\rho V_s L$ for vertical direction where ρ, V_s, V_p denote the mass density, shear wave velocity and compressive wave velocity, respectively, of the soil material outside the boundary (i.e. bedrock) and L si the tributary length of the corresponding node.*

2.2 Definition of seismic input

Input of an earthquake motion in RSL III 2D may be defined according with the definition of the base of the model. There are two typical cases (Fig. 3):

(a) ***A rigid base***, the bedrock is explicitly modelled and an acceleration time history is specified at the base of the model;

(b) ***A compliant base***, where a quiet (absorbing) boundary is used at the base of the model, and as a consequence, the bedrock is replaced with a dashpot and equivalent force, which defines the seismic input at the base of the computational soil domain.

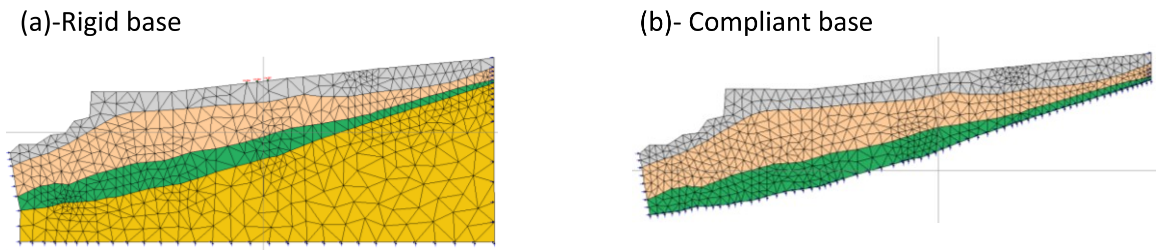


Figure 3: Seismic input and modelling: (a) *Rigid base*: explicit modelling of bedrock, prescribed vertical displacement and prescribed acceleration; (b) *Compliant base*: Bedrock replaced with dashpot (absorbing boundary) and equivalent seismic force.

2.2.1 Rigid base

When a rigid base is used (First approach) the seismic input is defined in terms *of prescribed acceleration* applied in the horizontal dirrection along the base of the bed-rock soil domain included in the finite element model (Additional Mass Method option as selected in the RSL-III-2D in the analysis panel-Figure 4).

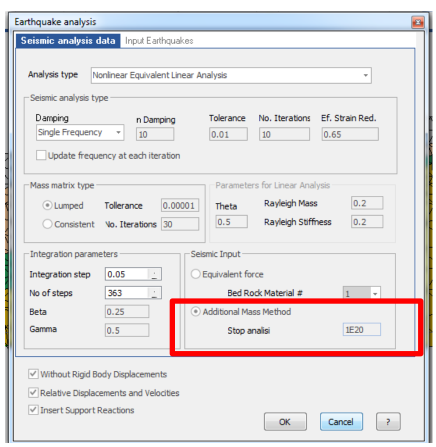


Figure 4: Seismic input for Rigid base modelling.

According to this approach [1] the response of the structure subjected to base excitation can be obtained by adding large masses along the horizontal degrees of freedom of the nodes at the bottom of the bedrock and by applying a force vector in the dirrection of mentioned degrees of freedom:

$$F(t) = \mathbf{M}_g^a \ddot{u}_g(t) \quad (12)$$

where $\mathbf{M}_g^a$ represents the diagonal matrix with consists with large values (penalty values defined in RSL-III-2D) added to the diagonal terms of the soil mass matrix along the degrees of freedom with prescribed accelerations and $\ddot{u}_g(t)$ is the recorded ground acceleration time series.

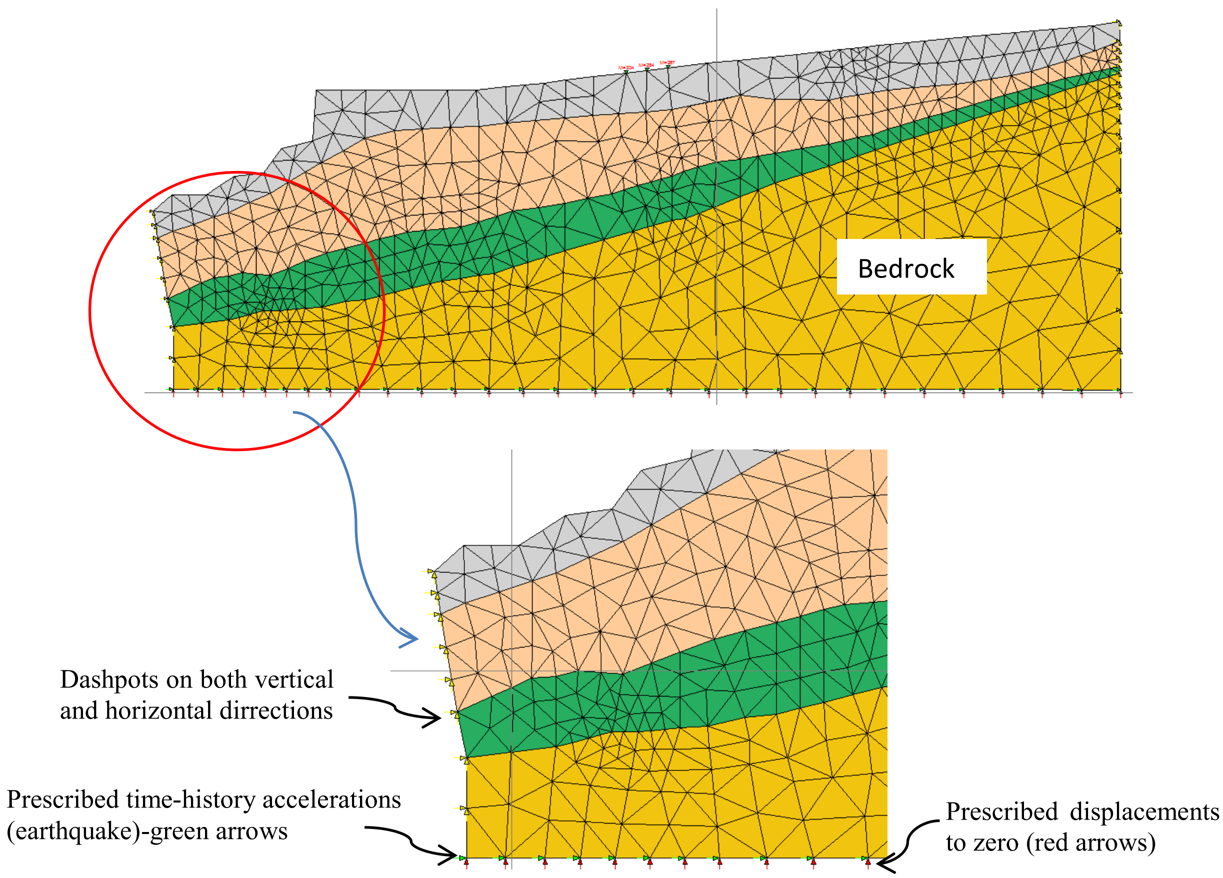


Figure 4: Dynamic modelling for rigid base

***Remark 1**: When this approach is selected the bed-rock region has to be modelled in the finite element model.*

2.2.2 Complaint base

When a compliant base (transmitting/absorbing base) is used, the input motion is a function of the material properties of the half-space below the finite element mesh (*bedrock region*), and the properties and geometry of the mesh. As we stated above on each node at the base and on the lateral boundaries of the soil domain are defined dashpots normal and tangential to the boundary respectively. This represent the lateral boundary condition when the input motion represents an outcrop acceleration, recorded at an outcrop of the half-space material. For a compliant base simulation an absorbing boundary must be specified along the base of the model by means of using the viscous boundary scheme as detailed in section 2.1 of the present manual. These dashpots of the quiet bounday absrob downward propagating waves so that they are not reflected back into the model. At these boundaries an acceleration time histroy cannot be inout directly (as in the previous case of Rigid base) because the boundary must be able to move freely to absorb incoming waves. The acceleartion-time history is transformed into a stress-time history for input. *In this approach the seismic input is defined in terms of equivalent nodal forces (or effective earthquake forces), which are proportional to the velocity of the incident wave, applied in the horizontal dirrection along the base of the soil domain included in the finite element model* (Equivalent force option as selected in the RSL-III-2D in the analysis panel-see Fig. 5).

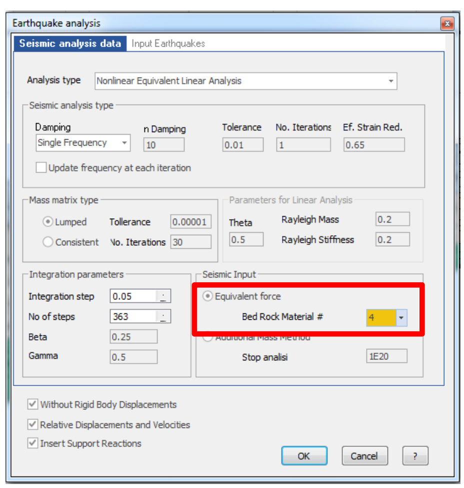


Figure 5: Seismic input for Compliant base modelling.

This outcrop input is applied as a shear force history, $F(t)$, as shown in Fig. 6. This creates an incident wave that is reflected back into the soil domain at the surface. The already mentioned dampers sown in Fig. 2 absorb the reflected wave simulating an infinite soil domain:

$$F(t) = \rho V_s A v(t) \tag{13}$$

where ρ is the density of the bedrock, V_s represents S wave velocity of the bedrock, $v(t)$ represents the seismic velocity excitation from the earthquake acceleration and A represents the tributary area associated to the loaded node.

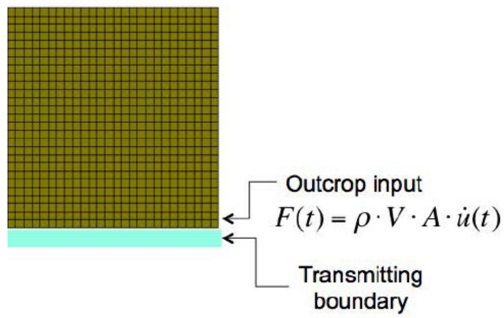


Figure 6: Compliant base: Definition of the seismic input. Equivalent force approach.

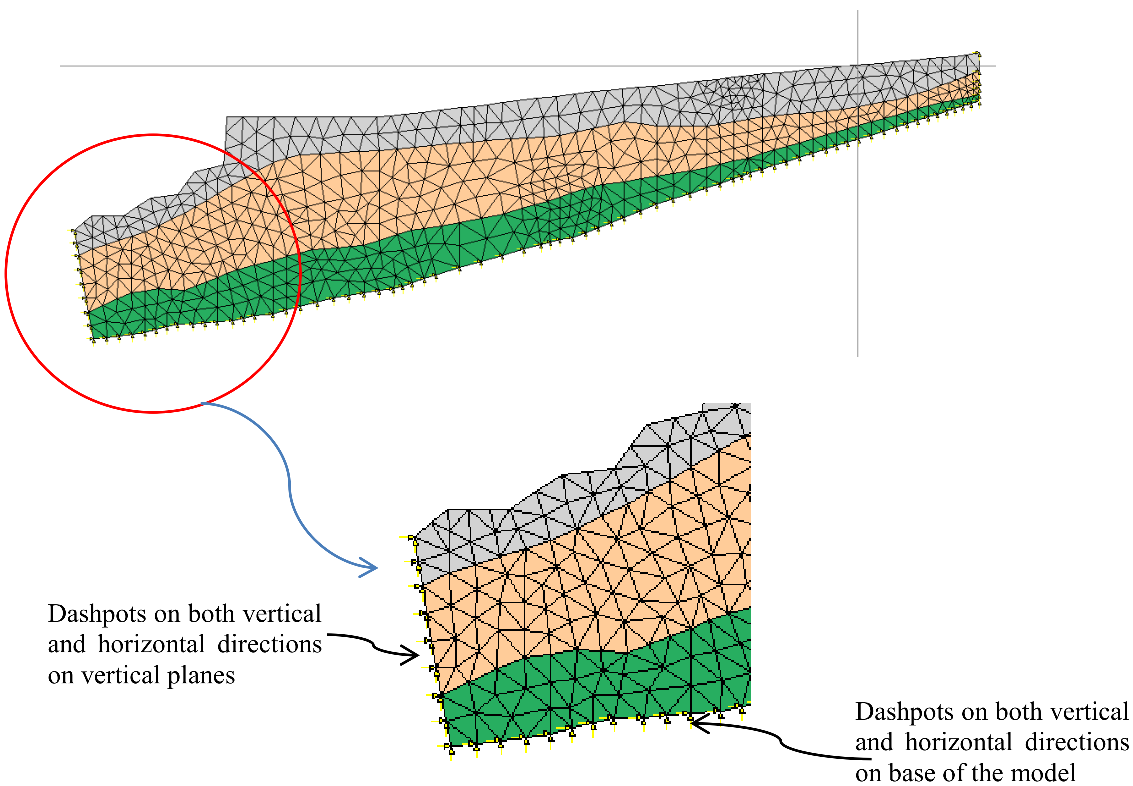


Figure 7: Dynamic modelling for compliant base

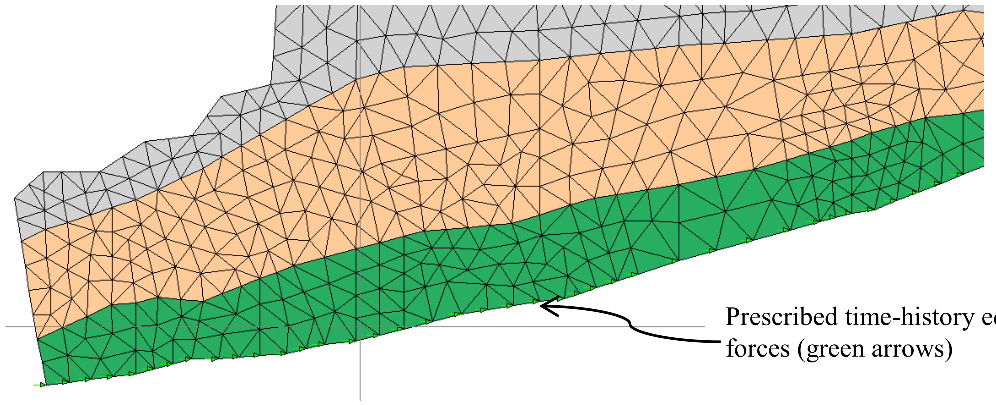


Figure 8: Seismic input for compliant base.

Remark 1: *A direct approach to acquiring the velocity from the earthquake acceleration by assuming zero initial conditions is applied. The velocity time series are determined as:*

$$v(t) = \int_0^t \ddot{u}_g(\tau) d\tau \tag{14}$$

where $\ddot{u}_g(\tau)$ is the recorded ground acceleration time series (out-crop acceleration). The acceleration data is baseline-corrected using the least-square curve fitting technique in order to reduce the drift in velocities and displacements.

Remark 2*: When this approach is selected by the user the bed-rock region is not necessary to be modelled. The seismic input will be defined at the bottom nodes of the model taking into account only the material properties of the bed-rock as stated above. Besides the vertical supports along the base line may be completely eliminated and replaced with the equivalent reactions as will be briefly described at the point 2.3 of the present manual.*

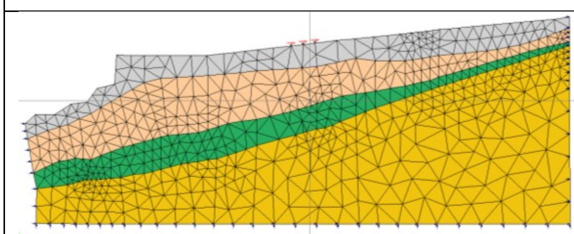
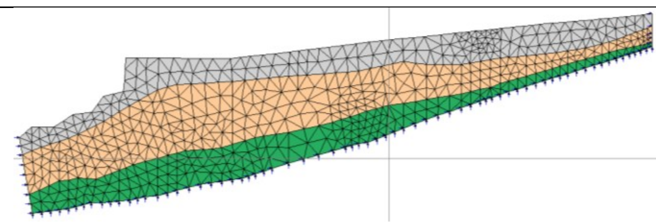
2.3. Gravity and dynamic analysis. Modelling.

A rigurous dynamic analysis is preceded by a static garvity analysis within a stage-wise procedure as will be briefly described next. In order to set-up the model such that both gravitational and dynamic behaviour of the soil to be revealed, we need to proceed in several steps to apply the static and dynamic boundary conditions.

Stage 1-Gravitational analysis

First we fix the base and lateral boundaries of the soil domain, set the various soil constitutive models to be linear elastic and apply gravity (Stage 1 in the RSL-III-2D). The base is restrained in both horizontal and vertical dirrections, the vertical boundaries are restrained only in the horizontal dirrection and are kept free in the vertical dirrection. The material of the soils is considered to behave elastically and the gravity loads are applied. The state of strain a and stress are determined and also an eigenvalue analysis is perfromed in order to determine the dynamic charcatersitics of the model (*see Remark of the section 2 of the present manual*).

Table 1: Boundary conditions-Stage 1

Rigid base	Compliant base
	

Stage 2-Dynamic analysis

In the second step, all the displacement constraints along the boundaries of the soil domain are removed and replaced with the corresponding support reactions (Table 2) recorded at the end of the first step-stage 1 (Insert Support Reactions option in the RSL-III-2D in the analysis panel-See Fig. 5).

Table 2: Boundary conditions (Reactions)-Stage 2

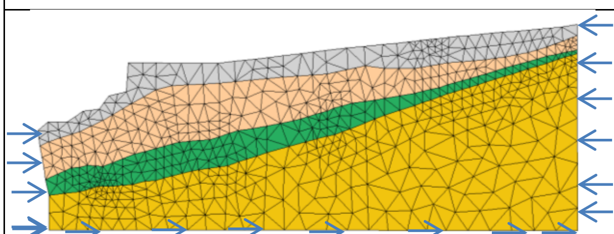
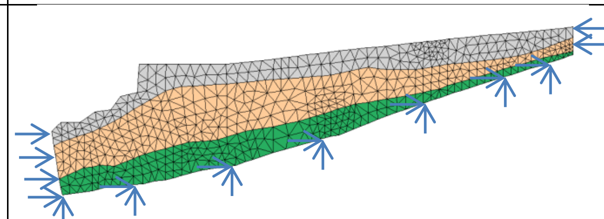
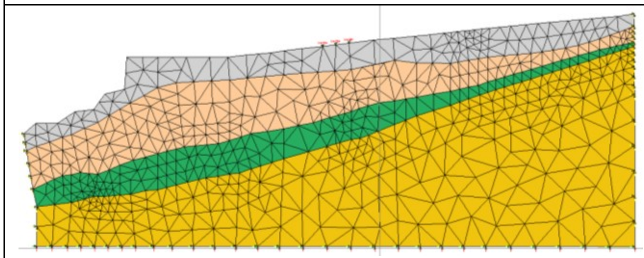
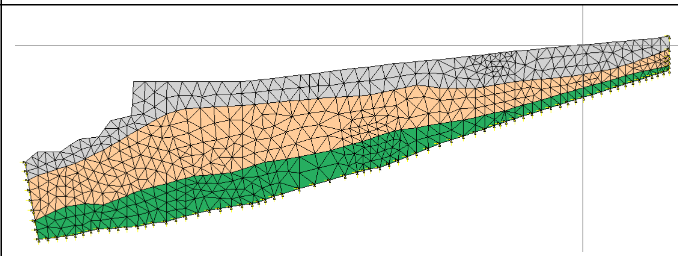
Rigid base	Compliant base
	

Table 3: Boundary conditions(dashpots) -Stage 1

Rigid base	Compliant base
	

After balancing the internal and external forces, dashpots in both the horizontal and vertical dirrections are added to the lateral boundaries of the soil domain to model the L-K transmitting boundaries discussed above (Table 3). For the rigid base at the base of the model prescribed displacements to zero are applied in order to restrain the vertical dispacement. For the compliant base along the base of the model L-K boundaries are asigned in the horizontal as well in the vertical dirrections.

Once the boundary conditions have been succesfully applied, the model is subjected to seismic excitation that is applied either as prescribed time-histroy acceleration in the case of rigid base or in the form of equivalent nodal forces defined earlier (Stage 2 in the RSL-III-2D). In both situations the equation of motion may be described as in Eq. 15:

$$\mathbf{M}\ddot{\mathbf{u}}+\mathbf{C}\dot{\mathbf{u}}+\mathbf{K}\,\mathbf{u}=\mathbf{F}(t)+\mathbf{F}_v \tag{15}$$

where **M** is mass matrix, **C** is damping matrix, **K** is stiffness matrix, **ü** is nodal acceleration vector, **u** is nodal velocity vector, **u** is nodal displacement vector assembled for the entire structure, **F**(*t*) represents the input nodal force vector defined either as in Eq. (12) for rigid base or as in Eq. (13) for compliant base, $\mathbf{F}_v$ is the force vector assigned at viscous boundaries during the gravity analysis (reactions recorded at the end of the first stage).

3. Material properties

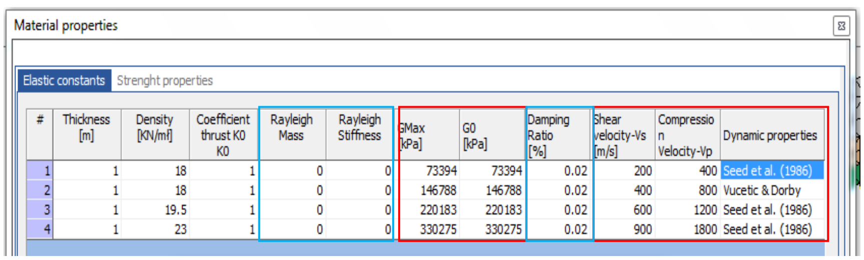
The dynamic soil properties that are needed in a ground response analysis are the small strain shear and normal wave velocity, V_s and V_p, shear modulus at low strain, G_{max}, and G/G_{max}–γ_{eff} and ξ–γ_{eff} curves describing the degradation of soil shear stiffness and damping with increasing amplitude of the effective shear strain, γ_{eff} (marked in red boxes in Fig. 4). Here ξ repressnts the damping ratio used in the computation of the damping matrices for each soil finite element. The shear modulus, G *(G_0 in the RSL-III-2D)*, can be determined from the measured shear wave velocities, V_s, i.e.,

$$G = V_s^2 \gamma_s / g \tag{16}$$

where, γ_s, is soil unit weight. Young's modulus can be then determined from the following relationship:

$$E = 2G(1+\nu) \tag{17}$$

where υ represents the Poissons's ratio. The $G/G_{max} - \gamma_{eff}$ and $\xi - \gamma_{eff}$ curves describing the degradation of soil shear stiffness and damping can be selected from the Material properties panel in RSL-III-2D program as described in Fig. 4. The stiffness degradation curve of the soil layers and the change in damping ratio with cyclic shear strain is illustrated in Fig. 5.


Material properties

Elastic constants | Strenght properties

#	Thickness [m]	Density [KN/m³]	Coefficient thrust K0 K0	Rayleigh Mass	Rayleigh Stiffness	GMax [kPa]	G0 [kPa]	Damping Ratio [%]	Shear velocity-Vs [m/s]	Compression Velocity-Vp	Dynamic properties
1	1	18	1	0	0	73394	73394	0.02	200	400	Seed et al. (1986)
2	1	18	1	0	0	146788	146788	0.02	400	800	Vucetic & Dorby
3	1	19.5	1	0	0	220183	220183	0.02	600	1200	Seed et al. (1986)
4	1	23	1	0	0	330275	330275	0.02	900	1800	Seed et al. (1986)

Figure 4: Material properties for dynamic analysis.

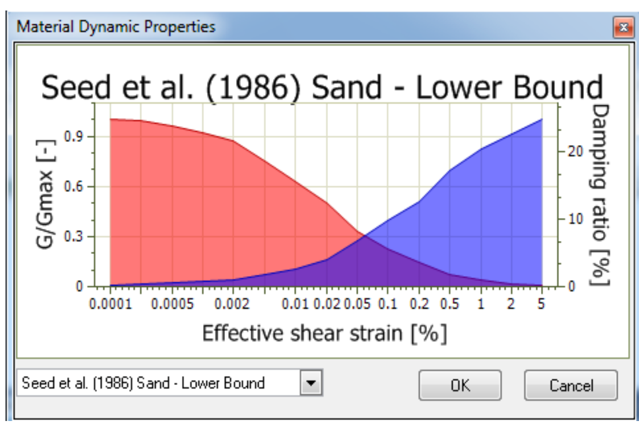


Figure 5: Stiffness degradation and damping ratio curves for soils.

***Remark 1**: The Rayleigh mass and stiffness parameters together with the damping ratio marked in blue boxex in Fig. 4 are required only when an Linear Dynamic Analysis with variable damping is selected to be performed.*

Remark 2: *It is important to pay attention at the correspondence between the elastic constants and wave velocities that defines the behaviour of soils. Eqs. (11) and (16)-(17) shows such a relationships.*

4. Main results: response spectrum for an earthquake

The response spectrum is calculated in any node of the mesh as the response of a SDOF systems in terms of accelerations, characterized by different stiffness k but same damping ratio ξ and subjected to the same earthquake. The information about the "structure" stiffness k is related with the natural period T reported on the X-axis of the response spectrum (Fig. 6). A direct integration of the equation of motion for a SDOF system is generaly used to compute the maximum acceleration for different SDOF systems for determining the response spectra. In the RSL-III-2D is implemented the exact *integration method of piecwise linear functions [1]*. This approach could be considered exact since the acceleration time-history obtained in the selected node is defined at regular time intervals and the linear interpolation is used between the data points. The procedure takes into account the fact that the *acceleration time-history* were obtained at regular time intervals Δt. The input data consists of the acceleration time history with duration t_f, the time interval Δt, the range for which the spectrum is to be computed (the response spectra are computed for periods of structure between 0.01s ans 10s on logarithmic scale) ten number of spectral values to calculate and the damping ratio (Fig. 6).

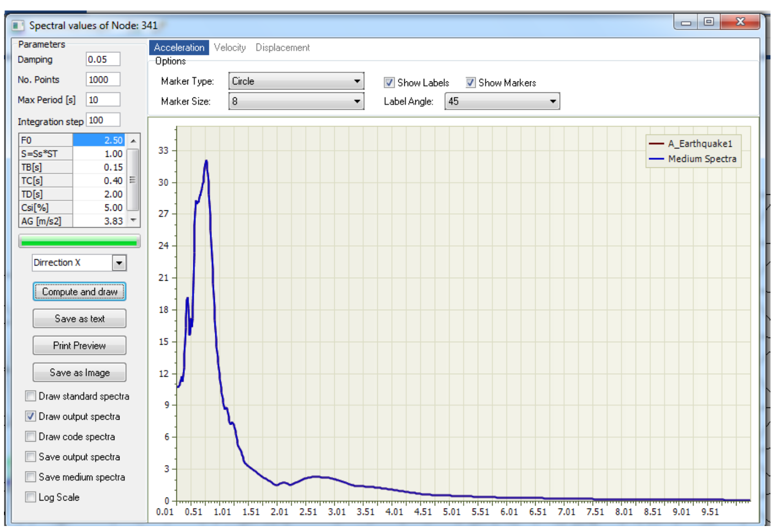


Figure 6: Response spectrum

5. References

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2. Mejia, L.H., Dawson E.M., Earthquake deconvolution for FLAC, 2006.

3. Zhang Y., Conte J., Yang Z., Elgamal A., Bielak J., Acero G., Two-dimensional nonlinear earthquake response analysis of a bridge-foundation-ground system, Earthquake Spectra, 2008.